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Monopoles in String Theory

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ABSTRACT

A realization of E_{n+1} monopoles in string theory is given. The NS five brane stuck to an Orientifold eight plane is identified as the 't Hooft Polyakov monopole. Correspondingly, the moduli space of many such NS branes is identified with the moduli space of $SU(2)$ monopoles. These monopoles transform in the spinor representation of an $SO(2n)$ gauge group when n D8 branes are stacked upon the orientifold plane. This leads to a realization of E_{n+1} monopole moduli spaces. Charge conservation leads to a dynamical effect which does not allow the NS branes to leave the orientifold plane. This suggests that the monopole moduli space is smooth for $n < 8$. Odd $n > 8$ obeys a similar condition. Using a chain of dualities, we also connect our system to an Heterotic background with Kaluza-Klein monopoles.

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1 Introduction

String theory has many realizations of the monopole solutions of 't Hooft and Polyakov. Basically, whenever there is an Abelian gauge field which has the potential of being a subgroup of a non-Abelian gauge group then we would expect an object which will be the analog of the monopole solution in field theory.

D branes offer a convenient framework for realizing monopole configurations in supersymmetric gauge theories and studying their moduli space. In a standard construction [?, ?, ?], monopoles for a non-Abelian gauge theory living on the world-volume of a set of branes are identified with other branes, of different type, ending on them. The type and the extension in space-time of the branes are chosen in order to have a BPS configuration. An overview of the various possibilities is described in Section 2. $SU(N)$ monopoles of magnetic charge k are easily described in this context and orthogonal and symplectic groups can be studied by introducing orientifold planes. Much more difficult is the study of E_n groups. In this paper we provide an explicit example of brane configurations that are naturally interpreted as E_n BPS monopoles.

E_n gauge groups, where n runs from 1 to 8, can be described in the Type I$'$ string theory using backgrounds where the dilaton is blowing up at some orientifold plane. These backgrounds with E_n symmetry are in one-to-one correspondence with the point in moduli space of the nine-dimensional Heterotic theory where there is a perturbative enhanced symmetry. The spectrum of electrically charged states in these Type I$'$ models has been extensively discussed in the literature, since it provides a non trivial check of string dualities. In particular, D0 branes, stuck on the orientifold plane, have been identified with the electrically charged states that become massless at the point in moduli space where an $SO(2n) \times U(1)$ gauge symmetry realized on D8 branes and Abelian bulk fields is enhanced to E_{n+1}. Much less attention has been paid to magnetically charged objects. We will show that stuck NS branes are naturally interpreted as monopoles. Together with systems of D6 branes stretched between the D8 branes responsible for the $SO(2n)$ symmetry, these NS branes give a stringy description of E_{n+1} monopoles.

In the absence of D8 branes, we obtain a description of k $SU(2)$ monopoles in terms of k NS branes moving on the orientifold plane. The moduli space of such monopoles is known to be smooth. Our NS branes are, by construction, stuck on the orientifold plane. For living outside the plane, a NS brane needs an image under the orientifold projection and, in principle, two NS branes could meet and move outside in the bulk. An obvious singularity would show up in the moduli space of the NS brane if they could leave the orientifold plane. We will see that a charge conservation argument does not allow them to leave. This is the string theory explanation for the smoothness of the monopole moduli space. The very same construction, generalized to the presence of D8 branes, predicts that the E_{n+1} monopole moduli space is also smooth.

The existence of monopoles in string theory can be also addressed from a different perspective. They can show up quite naturally in string compactifications. Sen realized that KK monopoles in the Heterotic string can be identified with BPS monopoles of a spontaneously broken gauge symmetry [?]. The same system was recently studied in order to get the exact moduli space of the Heterotic string near an ALE singularity without gauge field [?]. We will see that these Heterotic configurations are connected with the ones discussed in this paper by a chain of dualities. This connection is a further evidence of our identification of stuck NS branes with monopoles. On the other side, our construction can be used to explain and generalize the result in [?, ?]. Various papers have appeared which tried to connect the Heterotic moduli space with the Coulomb branch of three-dimensional N=4 gauge theories [?, ?, ?]. This connection is expected due to the relation between the Coulomb branch of some three-dimensional N=4 gauge theories and monopole moduli spaces [?]. Our point of view in this paper is that we look for the existence and the identification of the relevant monopoles in the problem, leaving three dimensional gauge theories aside.

This paper is organized as follows. In Section 2, we give an overview of the various string theory realizations of monopoles. Section 3 contains the description of the Type I′ configuration that is the object of this paper and the explicit realization of the E_{n+1} monopoles. Section 4 makes a connection with the KK monopoles in the Heterotic string. Various comments on the smoothness of moduli space are contained in Section 5. Section 6 contains a brief discussion of the D_k case.

2 Monopoles in String Theory

Examples of monopoles in string theory have been studied in great detail and here we summarize some of the cases.

Probably the most studied example is the one realized by a D1 brane which is stretched between a pair of two D3 branes [?, ?, ?, ?, ?]. This is the classical example for a spontaneously broken four dimensional N=4 supersymmetric YM theory with gauge group SU(2), the one which was explicitly used in the solutions of 't Hooft and Polyakov. There are several natural realizations of such monopole solutions. A Dp brane stretched between a pair of Dp+2 branes will be a $p-1$ brane solution in a $p+3$ dimensional theory, for $p \leq 6$. The classical solution of the Higgs field represents the shape of this brane configuration [?]. This family of solutions is related to the case $p=1$ by a set of T-dualities. The field theory interpretation of such dualities is dimensional reduction starting from a higher dimensional theory and reducing some of its dimensions. The case $p=0$ is somewhat special and represents an object which has the interpretation of an instanton in 3 dimensions. It is identified with the Euclidean monopole solution of Polyakov [?] and is represented by a Euclidean D0 brane.

Another example of monopoles in string theory is realized by applying S-duality to the case $p=3$. This gives a configuration of a D3 brane stretched between a pair of NS five branes, a configuration which was well studied [?]. One can apply a further SL(2,Z) transformation on this configuration and get a D3 brane stretched between a pair of (p,q) five branes. [?] also demonstrated a connection to three dimensional gauge theories with N=4 supersymmetry by mapping the Coulomb branch moduli space to certain monopole moduli spaces, a problem which was motivated by field theory studies, for the $SU(2)$ case in [?] and for $SU(n)$ in [?].

One other example is that of [?] in the study of the Heterotic string on a Taub-NUT space. Sen finds that the monopole solution in the form of a Kaluza Klein monopole is interpreted as a 't Hooft Polyakov monopole. The radius of the circle in the Taub-NUT space plays the role of the scalar VEV in the spontaneously broken $SU(2)$ group which becomes enhanced when the radius approaches the self dual radius.

A common feature to all of these string theory configurations is the fact that they all represent classical monopole solutions. Correspondingly the moduli space of solutions for these objects coincides with the moduli space of monopoles [?]. The structure of the moduli space solution for the simplest case of 2 monopoles in an $SU(2)$ gauge theory was studied in detail in the book of Atiyah and Hitchin. The space is known as the Atiyah Hitchin space. It has dimension 4 and admits a hyperKhler metric. We intuitively identify the 4 parameters as the 3 space positions of the monopole and a phase angle associated with large gauge transformations. The Atiyah Hitchin metric consists of the following three types of contributions. There is a natural expansion parameter for the monopole moduli space. It is given by the distance between the two monopoles as measured in units of inverse scalar VEV. A typical metric component has an expansion of the form [**?**]

$$1 - \frac{\text{const}}{\langle\phi\rangle x} + O(e^{-\langle\phi\rangle x}) \tag{1}$$

where x is the monopole separation and $\langle\phi\rangle$ is the Higgs VEV. For very large separations the moduli space looks flat with two orbifold singularities. There is a correction to this metric coming with an inverse power of the separation between the monopoles. This changes the space to a circle bundle over R^3. There are further exponential corrections to the metric which make this space smooth.

It is interesting to consider how such exponential corrections arise in each of the examples discussed above. We will mention the corrections for the moduli space of two $SU(2)$ monopoles in each of these cases. For a pair of Dp branes stretched between a pair of Dp+2 branes one can stretch a fundamental string which is bounded by both the Dp and the Dp+2 branes. This gives rise to a worldsheet instanton with an action $x\phi$ where x is the separation between the two Dp branes interpreted as the separation between the two monopoles and ϕ is the scalar VEV of the spontaneously broken $SU(2)$, measured as the distance between the two Dp+2 branes in units of l_s^2.

The case of a pair of D3 branes stretched between two NS branes gives rise to a Euclidean D1 brane bounded between both the D3 branes and the NS branes. Here ϕ is the distance between the two NS branes measured in units of $g_s l_s^2$. Similarly for a pair of D3 branes stretched between a pair of (p, q) five branes the exponential correction is due to a (p, q) string bounded by both the three branes and the five branes. ϕ is the distance between the two (p, q) five branes measured in units of the (p, q) string tension.

The purpose of this paper is to discuss a certain class of Type I$'$ theories. In the spirit of the introduction above, whenever we observe a gauge group we would like to look for the monopole solutions for this theory. One guide line for this search will be what we call a "generalized Montonen Olive duality." This duality in one of its forms states that the monopole spectrum of a four dimensional N=4 supersymmetric gauge theory with gauge group G sits on the lattice of the electric spectrum of another four dimensional N=4 supersymmetric gauge theory with gauge group $\widetilde{G}$. $\widetilde{G}$ is defined as having a root lattice which is the dual of the root lattice of G. The extension of the Montonen Olive duality that we will use is in dimensions different than 4. We will say that the monopole spectrum of a gauge theory with 16 supercharges in $p{+}3$ dimensions and a gauge group G sits on the same lattice as the electric spectrum for a gauge theory with 16 supercharges in $p+3$ dimensions and gauge group $\widetilde{G}$. This property will be used when discussing some particular backgrounds with various groups G.

3 Monopoles in Type I′ String Theory

Type I′ in its most simple form is described by a background with $R^9 \times S^1/Z_2$ where the Z_2 acts on the circle by reflection of the coordinate while changing the orientation of the string worldsheet. It is convenient to think of the circle as an interval. There are two fixed planes under the Z_2 action which carry a D8 RR charge of magnitude -8. These planes are called orientifold planes and are denoted by $O8^-$. Charge conservation on the interval implies a constraint on the number of physical D8 branes, which carry RR charge 1, to be 16. There is an option to replace one of the orientifolds by a positively charged orientifold, denoted by $O8^+$, with RR charge +8. In this case charge conservation on the interval implies that there are no physical branes in the bulk of the interval. The second option, having no physical D8 branes, will be less interesting for the applications in this paper and will not be considered.

The Type I′ string coupling, or more precisely its inverse, is a varying function on the positions of the various D8 branes. It satisfies a Laplace equation in 1 space dimension with delta function sources at the positions of the D8 branes on the interval, together with the negative charge source at the boundaries of the interval. As such it is a piece-wise smooth linear function. Denote the positions of the D8 branes on the interval by x_i and the coordinate on the interval by x, then the string coupling obeys the following equation

$$\frac{l_s}{g_s(x)} = \sum_{i=1}^{16} |x - x_i| - 8|x| - 8|R - x| + \frac{l_s}{g_s^0}. \tag{2}$$

When this function vanishes the string coupling diverges and some states in the string spectrum become massless. Such a vanishing can happen only at the boundaries of the interval since it is a positive function. As discussed in

[?] a half [3] D0 brane which is stuck to one of the orientifold points becomes massless as the string coupling diverges. This state together with a massless half anti D0 brane and the gauge field which sits in the string coupling multiplet form a gauge group $SU(2)$. One can describe this process as an inverse Higgs mechanism for getting an enhanced $SU(2)$ gauge group. The scalar VEV is given by the D0 brane mass,

$$\langle\phi\rangle = \frac{1}{2g_s l_s}. \tag{3}$$

The gauge coupling of this gauge theory is given by the usual coupling on a D8 brane,

$$\frac{1}{g_{YM}^2} = \frac{1}{g_s l_s^5}. \tag{4}$$

At this point we would like to ask what is the object which can be identified with the 't Hooft Polyakov monopole for this spontaneously broken $SU(2)$. Since it is a BPS state it is enough to know its mass in order to identify its charge. On the other hand we can apply the classical formula for the tension of the monopole, T_{mon} from its field theory value,

$$T_{mon} = \frac{\langle\phi\rangle}{g_{YM}^2} = \frac{1}{2g_s l_s^6}. \tag{5}$$

This formula identifies the monopole as a half NS brane which is stuck to the $O8^-$ plane.[4] More details on this identification appear in section 4.

The system of NS branes and D8 branes is the same system which was studied in the context of six dimensional gauge theories in [?] and [?]. For definiteness we will take the NS branes to lie along 012345 directions, the D8 to be point like in direction 6 and, when present, D6 branes will span the coordinates 0123456.

This picture can be generalized to include additional physical D8 branes sitting at the $O8^-$ plane. For finite string coupling the world volume theory of n D8 branes stacked at the orientifold plane is $SO(2n)$. Together with the

[3] Half a brane in this case means that the brane carries half of the charge of a physical brane. Away from the orientifold a physical brane consists of a half brane and its image under space reflection. On the orientifold, half a brane can exist with no images.

[4] One may question the existence of half NS brane on an $O8^-$ plane. We thank Oren Bergman for raising this issue. However, its existence is imposed by the "generalized Montonen Olive duality" principle.

$U(1)$ gauge field, which is in the multiplet which contains the dilaton, this symmetry is enhanced to E_{n+1} for a diverging dilaton at the fixed point of the interval. We may ask where are the extra states in the adjoint representation of the E_{n+1} theory. This question was analyzed by [?] who argued that a half D0 brane stuck at the $O8^-$ plane transforms under the spinor representation of $SO(2n)$ and is naturally charged with respect to the $U(1)$ with a charge $\frac{1}{2}$. It is amusing to note that such identification requires a non-trivial bound state of two half D0 branes for the cases E_7 and E_8 [?]. To find this bound state is a non-trivial problem in quantum mechanics of such stuck D0 branes.

With this picture in mind we now turn to the spectrum of monopoles in such an E_{n+1} theory. At this point we recall the "generalized Montonen Olive duality" principle which was mentioned at the previous section. As the E_{n+1} root lattice is self dual, the monopole spectrum sits in the adjoint representation of the E_{n+1} group. This decomposes naturally to monopoles in the adjoint representation of the $SO(2n)$ group and neutral under the $U(1)$ and to monopoles in the spinor representation of $SO(2n)$ charged under the $U(1)$. The BPS objects in the adjoint representation of $SO(2n)$ are monopoles for the D8 branes and are naturally given by $D6$ branes stretched between a pair of neighboring D8 branes. This is one of the cases which was mentioned in the previous section for $p = 6$. What about the BPS objects in the spinor representation? The $U(1)$ charge of the object and its mass formula lead to identify it with a half NS five brane stuck at the $O8^-$ plane. We may ask why it transforms in the spinor representation of $SO(2n)$. This is not a simple problem and we may only give some suggestions. According to [?] there is a linking number [?] which is induced by the D8 brane on the stuck NS brane. This linking number is half the unit of charge for a D6 brane. One may ask if there is a state associated with this charge. It must be a supersymmetric singlet as there are no additional massless multiplets which are induced by the D8 brane on the NS brane. It is highly suggestive that quantization of such objects leads the NS brane to transform under the spinor representation of $SO(2n)$. It is not easy to show this, though. It is again amusing to note that for the cases of E_7 and E_8 groups the analysis above suggests that there will be non-trivial bound states of a pair of half NS branes which are required to complete the adjoint representation of the gauge group. It is an interesting problem to try and show how this arises.

3.1 Smoothness Puzzle

Let us look more carefully at the case of a gauge group E_1. This corresponds to an $O8^-$ plane with a diverging string coupling at the enhanced point and a finite string coupling for the spontaneously broken phase. Let us take two half NS branes stuck on the $O8^-$ plane. According to our identification these two branes are two monopoles in an $SU(2)$ gauge group. Correspondingly their moduli space, which consists of 3 relative positions inside the $O8^-$ plane and a relative phase angle in the eleventh direction, is identified as the (reduced) moduli space of 2 $SU(2)$ monopoles or the Atiyah Hitchin manifold. The manifold is known to be smooth. This points to some difficulty. There is apparently a singularity in the moduli space of two half NS branes as they move inside an $O8^-$ plane. They can approach each other along the three directions inside the $O8^-$ plane and leave to the bulk of the interval as a pair of a brane and its image under the reflection of the Type I′ interval. The motion away from the orientifold plane corresponds to a scalar in a massless tensor multiplet. This tensor multiplet does not exist as a massless state in the phase in which the NS branes are stuck to the orientifold. Correspondingly, there is a singular point in the hypermultiplet moduli space in which the tensor multiplet becomes massless. This is in contradiction with the smoothness of the moduli space of two monopoles!

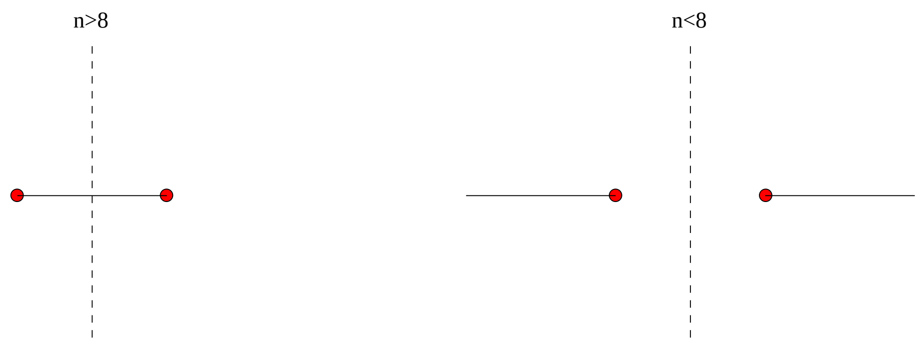

Figure 1: A physical NS brane in the presence of an $O8^-$ plane. The dashed line represents an $O8^-$ plane with a stack of n D8 branes and the circle denotes half a physical NS brane. The solid lines are D6 branes. There are two cases to consider depending on the number n. For $n > 8$ the D6 branes stretch to the $O8^-$ plane while for $n < 8$ they stretch away from it.

How can this be solved? We recall that the Type I′ has irregularities in the form of a cosmological constant in an intermediate region between two D8 branes or between D8 brane and an $O8^-$ plane. For our example of the E_1 theory the cosmological constant outside the $O8^-$ plane is -8. As discussed in [?], a NS brane in a non-zero cosmological constant background has D6 brane tails. For the case of -8 we have 8 half D6 branes stretching away from the $O8^-$ plane, as in figure 1 for the case $n = 0$. It is crucial that the D6 branes are stretched away from the $O8^-$ plane. This is the key point which resolves our puzzle. Suppose that a pair of stuck half NS branes meet and attempt to move away from the $O8^-$ plane. This is not possible due to energetic reasons. Long D6 branes can not be formed as soon as the NS branes leave to the bulk of the interval. We conclude that the pair of half NS branes can not leave the $O8^-$ plane. Instead they are confined to the plane and there are no singular points associated to the motion outside. The moduli space in question is smoothed out. In some sense this effect can be thought of as a higher order effect. The classical moduli space of two half NS branes has a flat metric with an orbifold singularity,

$$\frac{R^3 \times S^1}{Z_2}. \tag{6}$$

The singularity at the origin is interpreted as the point at which naively a pair of half NS branes meet and attempt to leave the $O8^-$ plane. This singularity is smoothed out by an exponential correction to the metric coming from Euclidean D0 branes, which are stretched between the two half NS branes, of action $\frac{x}{g_s l_s}$. Here x is the distance between the NS branes, g_s is the string coupling at the $O8^-$ plane and l_s is the string scale. These objects prevent the NS branes from meeting on the $O8^-$ plane. We can restate this in terms of the mass for a tensor multiplet. Classically, there is a massless tensor multiplet which arises as the NS branes meet and leave the $O8^-$ plane. This tensor multiplet gets exponential contributions to its mass from instantons coming from Euclidean D0 branes stretched between the two half NS branes.

It is easy to generalize this picture to a higher number, n, of D8 branes sitting on the $O8^-$ plane. As long as $n < 8$ in figure 1 the charge of the combined system, $O8^-$ and n D8 branes, is negative. Consequently, half physical NS branes are confined to this system and will not leave to the bulk of the interval as a pair.

The following statement can be used as a prediction on the behaviour of the moduli space of monopoles for several gauge groups. Let us first consider the case E_1, namely an $O8^-$ plane. k half NS branes stuck on this plane correspond to k $SU(2)$ monopoles. Their moduli space is identified with the moduli space of k $SU(2)$ monopoles. It is a $4k$ dimensional space which is believed to be smooth. (There is a 4 dimensional trivial part which is associated to the center of mass motion for the monopoles but the remaining space is believed to be smooth). The case $k = 2$ was calculated explicitly by Atiyah and Hitchin and was shown to be smooth. Our resolution to the puzzle actually supports the claim that the k monopole moduli space is smooth for any k.

We can further extend this claim for E_{n+1} models. Here the statement will only deal with a particular part in monopole moduli space of E_{n+1} which consists only of those monopoles in the spinor representation of $SO(2n)$ under the decomposition $SO(2n) \times U(1) \subset E_{n+1}$. The moduli space of k such monopoles is smooth for any k. There are no singularities associated with two such monopoles meeting and leaving to the bulk of the interval.

3.2 $n > 8$ and an Odd Puzzle

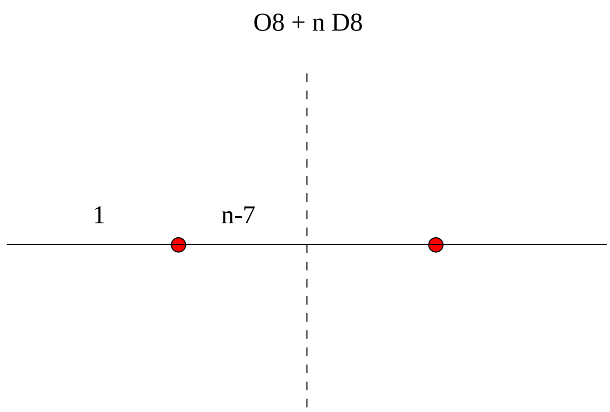

Figure 2: A physical NS brane in the presence of an $O8^-$ plane and an odd number, n of D8 branes. There are $n - 7$ half D6 branes stretching between the half NS brane and its image. There is a half D6 brane stretching away from the half NS brane.

What about the case of n D8 branes that are enough to make the charge of the $O8^-$ plane positive? In this case a pair of half NS branes is allowed to leave the plane and a total of $n - 8$ half D6 branes can stretch in between the two half NS branes. Here we face another puzzle which first appears for the case $n = 9$. As already pointed out in [?], consistency with tadpole cancellation restricts the number of D6 branes which cross the $O8^-$ plane to be even. This is not the case for $n - 8$ odd. One would conclude that $n > 8$ odd is not a consistent brane configuration. However, there is another alternative for a physical NS brane in the bulk which is consistent with this restriction. This is the alternative which was discussed in [?]. For n odd, one can stretch, as in figure 2, $n - 7$ half D6 branes in between the half NS brane and its image and a single half D6 brane away from the half NS brane. This configuration is consistent with charge conservation for the RR 6 brane charge in the presence of a non-zero cosmological constant.

A brane which is stretched away from the NS brane poses the same problem that we had for the $n < 8$ case. Suppose that in the background of n odd D8 branes stacked upon an $O8^-$ plane there is a pair of stuck half NS branes. Such a pair can not leave the $O8^-$ plane to the bulk of the interval. It is not allowed by the same energetic reasoning as in the case for $n < 8$. For even $n > 8$ there is no need to stretch a D6 brane away from the NS brane and therefore a pair of half NS branes is allowed to leave the $O8^-$ plane. It is not clear how to interpret this phenomenon from the point of view of monopoles in the corresponding gauge theory.

3.3 A massless Three Brane?

Singularities in moduli space are typically associated with the appearance of some massless states, or more generally tensionless objects. The classical moduli space of two half NS branes, (6), is singular. Here classical means with respect to the natural expansion parameter which is determined from the form of the Atiyah Hitchin manifold. In our case it is the Euclidean D0 brane action $\frac{x}{g_s l_s}$, with x the distance between the two NS branes. We may ask what is the object which becomes massless/tensionless in the limit of a small string coupling. It is easy to find that this object is given by a D4 brane which is stretched between the two NS branes [5]. This is the only object which can stretch between two NS branes inside an $O8^-$ plane in a supersymmetric fashion. For small expansion parameter the D4 brane gives rise to a tensionless three brane as the pair of half NS branes coincide. This is clearly a naive picture since the moduli space gets smooth by exponential

[5] One may actually need to consider a pair of half D4 branes in order to avoid problems of existence for such a configuration. The discussion is not affected by this, though.

corrections in the expansion parameter. Correspondingly the three brane gets exponentially small corrections to its tension of the order of $\exp(-\frac{r}{g_s l_s})$. This three brane actually never gets tensionless as the moduli space is smooth. It is amusing to note that this object maps in the Heterotic string language, which will be discussed in the next sections in detail, to a small instanton which wraps a vanishing two-cycle.

4 Heterotic String Moduli Space

In this Section, we compare the identification of E_{n+1} monopoles with configurations of D6/D8/NS branes by performing a series of T/S dualities. At the same time, we make contact with different approaches, where monopole moduli spaces appear in the perturbative description of the Heterotic string.

Starting with the configuration discussed in the previous Section, a T duality along the 6 direction brings us to the Type I theory. At this point an S duality leads to the Heterotic string, where the enhanced gauge symmetry phenomenon of the E_{n+1} theory can be perturbatively studied. In this process, the D8 branes are mapped to D9 branes in the Type I theory and the $SO(32)$ gauge fields in the Heterotic string, while the k NS branes are mapped into k Kaluza-Klein monopoles (this is the same as a Taub-NUT space) of the Type I or Heterotic theory. If the identification of the Type I′ stuck NS branes as monopoles is correct, we expect that the moduli space of monopoles appears as the space of vacua of the Heterotic string defined on a Taub-NUT space. It was shown in [**?**, **?**] that the moduli space of k $SU(2)$ monopoles indeed appears in this context. More precisely, Sen argued that the Kaluza Klein monopole is identified with a 't Hooft Polyakov monopole of the $SU(2)$ gauge group which is enhanced at the self dual radius. Studying the problem in the limit where the Taub-Nut space reduces to an ALE space, Witten conjectured a relation to three dimensional gauge theories. The connection between the two points of view is made by realizing that the monopole moduli space coincides with the Coulomb branch of the three dimensional gauge theory [**?**, **?**].

We can formulate the problem, from the point of view of the Heterotic string, as follows. We are interested in the moduli space of hypermultiplets in the Heterotic string on K_3. In order to simplify the problem and keep only few hypermultiplets, we replace K_3 with a non-compact space obtained by zooming on local singularities of the manifold. This is equivalent to consider the Heterotic string defined on an ALE space. This configuration can be continuously deformed to the one we obtained after T and S duality; a Taub-NUT space has topology $R^3 \times S^1$ and reduces indeed to an ALE space in the limit where the radius of the S^1 becomes large. As a result of the zoom, the 6 dimensional effective theory describing the moduli for this "compactification" is decoupled from gravity and bulk modes. We want to focus on the moduli space corresponding to the 6 dimensional modes. Other parameters, which specify the background, such as the radius of the Taub-NUT space or Wilson lines for the 10 dimensional gauge fields, are not dynamical, since they are VEVs of 10 dimensional decoupled fields. Due to the decoupling of gravity, the 6 dimensional hypermultiplet moduli space is a hyper-Kähler manifold. The hypermultiplet moduli space is not corrected by string loops, but receives α' corrections. The classical moduli space is generically singular. In backgrounds with non-trivial gauge bundles, some singularities, for example those associated with small instantons, survive quantum corrections and find their explanation in non-perturbative phenomena. In the trivial bundle case, there are examples where quantum corrections smoothes the classical singularity [?, ?]. In these examples, quantum corrections reproduce the expected moduli space of monopoles.

4.1 The moduli space of KK monopoles

Let us review what is known about the moduli space of KK monopoles in the Heterotic string with trivial gauge fields [?, ?]. These results strongly support our identification of stuck NS branes in the dual Type I$'$ theory as BPS monopoles.

Consider the dynamics of k Kaluza-Klein monopoles in the Heterotic theory, or, in other words, a multi-centered Taub-NUT space. Their exact moduli space has been found by Sen [?].

The Taub-Nut space has a metric

$$ds^2 = V(\vec{x})d\vec{x}^2 + V^{-1}(\vec{x})(dx_4 + \vec{\omega} \times d\vec{x})^2 \tag{7}$$

which is completely specified by the potential

$$V(\vec{x}) = 1 + \sum_{i=1}^{k} \frac{1}{|\vec{x} - \vec{x}_i|} \tag{8}$$

In the perturbative Heterotic string, the moduli space corresponding to this "compactification" is parameterized by the expectation values of $k-1$ six dimensional hypermultiplets. The scalar components of these hypermultiplets are obtained by reducing the metric and the B field along the $k-1$ two-cycles of the space. In the dual Type I′ description, these hypermultiplets live on the world-volume of the stuck NS branes and parameterize their position in the space transverse to it. The classical moduli space is a symmetric product of $k-1$ copies of $R^3 \times S^1$ and is singular. The singularity is the effect of considering the low energy supergravity; it is smoothed out by higher derivative corrections. This can be understood as follows [**?**]. By varying the radius R of the S^1, the $SO(32)$ Heterotic string can be driven to the self-dual point ($R^2 = \alpha'$) where there is a perturbative enhanced $SU(2)$ symmetry. The $W^{\pm}$ bosons responsible for enhancing the symmetry from $U(1)$ to $SU(2)$ are perturbative BPS string states with $(n, m) = \pm(1, -1)$ units of momentum and winding along S^1, as can be seen from the BPS formula

$$M_{BPS}^2 = (\frac{n}{R} + \frac{mR}{\alpha'})^2. \tag{9}$$

The Kaluza-Klein monopoles are identified as BPS monopoles of the $SU(2)$ enhanced symmetry. A KK monopole has indeed the same magnetic charge $(1, -1)$ with respect to the S^1 momentum and winding of a BPS monopole. That a KK monopole carries one unit of momentum is part of the definition of the object. That it also carries -1 unit of winding follows from the equation

$$dH = \mathrm{Tr}\, F \wedge F - \mathrm{Tr}\, R \wedge R \tag{10}$$

that needs to be satisfied in any consistent Heterotic model. It follows from this identification that the exact moduli space for k KK monopoles is the moduli space of k BPS monopoles of $SU(2)$. This is expected to be a smooth manifold; for $k = 2$ it is the Atiyah-Hitchin manifold.

Monopoles exist in the phase where the group is spontaneously broken to $U(1)$ by the expectation value of some Higgs field. The metric on the moduli space has an expansion in terms of the product $\langle\phi\rangle x$ of the monopole distance with the Higgs field VEV which is given in formula (1) of Section 2. We can identify the gauge theory parameters with the Heterotic ones as follows. The 9 dimensional $SU(2)$ group has coupling constant $Re^{-2\phi_h}$, where ϕ_h is the Heterotic Dilaton. It follows from formula (9) that the VEV of the field responsible for the spontaneous symmetry breaking is related to the S^1 radius by $R/\alpha' - 1/R$. We see that the expansion of metric components in equation (1) can be interpreted, in the large radius limit, as the

α'/R^2 expansion. For large R^2/α' we recover the supergravity result, a flat singular metric. The perturbative expansion only contains a one-loop (in α') contribution, which comes with a negative sign. Only after including the non-perturbative corrections due to instantons the metric becomes smooth.

A second example, due to Witten [**?**], deals with the moduli space of the Heterotic string near an ALE singularity with no gauge bundle. For a Z_k singularity, the relevant 6 dimensional fields are $k-1$ hypermultiplets parameterizing the blowing-up modes of the manifold. Combining symmetries, semi-classical arguments and the assumption of smoothness, the moduli space for $k=2$ was unambiguously identified in [**?**] with the Atiyah-Hitchin manifold. It was then conjectured that the moduli space for a singularity of type G is the Coulomb branch of a three dimensional gauge theory with 8 supercharges and gauge group G. Due to the relation between monopole moduli spaces and three dimensional gauge theories [**?**], this result agrees with Sen's one. This example can be indeed considered as a limit of the previous one, where the radius R is sent to infinity while keeping the blowing-up parameters fixed; this scaling preserves the form of the metric. Notice that the $SU(2)$ group is somehow hidden in this approach.

The expansion of the metric components in equation (1) in this example is just the α' expansion. There is a world-sheet one-loop correction to the classical singular metric and a series of world-sheet instanton corrections. Notice that a world-sheet instanton in this case is a fundamental string wrapped over a two-cycle of the ALE space. An S-duality transforms it into a D1 brane of the Type I theory and a T-duality into an Euclidean D0 brane stretched between the NS branes in the Type I′ description. This is in agreement with the identification made in Section 3.1 of D0 branes as responsible for the corrections to the classical metric. As discussed in section 3.3, we can here mention again the existence of a BPS three brane given by a small Heterotic instanton which wraps a two-cycle of the ALE space. Classically, when the metric is singular, it is tensionless. As one includes α' corrections the tension gets exponential corrections and does not vanish anywhere on the moduli space.

The question of smoothness of the moduli space for the Heterotic on ALE is not as clean as in the Type I′ picture. It is not clear how to translate the statement on charge conservation made in section 3.1. It is also not clear how to translate the statement that the NS branes are confined to the $O8^-$ plane to the Heterotic picture. Such questions, if answered, would give us a better perspective on the issue of smoothness of the moduli space from the

Heterotic string point of view.

4.2 Generalization to the E_{n+1} case

The generalization of Sen's example to the E_{n+1} case is simple. In the Heterotic language, we can tune the radius and the Wilson lines in such a way that an E_{n+1} symmetry in enhanced. The BPS formula, for generic Wilson lines, reads

$$M_{BPS}^2 = \left(\frac{n - A \cdot P - mA^2/2}{R} + \frac{mR}{\alpha'} \right)^2 \tag{11}$$

where A is a sixteen component vector representing the Wilson line and P is an element of the $SO(32)/Z_2$ lattice. In Type I$'$ picture, an entry 0 in the 16 component vector A corresponds to a D8 brane sitting at the orientifold plane where the coupling constant is blowing up, while an entry 1/2 corresponds to a D8 brane at the other orientifold. E_{n+1} is obtained by choosing a vector of the form $A = (0, 0, ..., 0, 1/2, ...1/2)$, with n entries equal to zero. The critical radius is $R^2 = \alpha'(1 - A^2/2) = \sqrt{\alpha'(8-n)/8}$. At the critical radius, the gauge symmetry $SO(2n) \times U(1)$ is enhanced to E_{n+1}. The electrically charged objects that are enhancing the gauge symmetry have typically non-zero winding and momentum, and quantum numbers under $SO(2n)$. The magnetically charged object with the same quantum numbers are identified as BPS configurations with $SO(2n)$ monopoles on a Taub-NUT space. As the radius R is taken to be large they become heavier and narrower as expected from their behavior as monopoles. We can trace what these objects are from the Type I$'$ picture. The spinor representation of $SO(2n)$ is given by a Taub NUT space where the order of the space corresponds to the monopole number and the values of the blow up parameters serve as positions of these monopoles. The adjoint representation of $SO(2n)$ is given by fractional small Heterotic instantons. Such unusual Heterotic background has a moduli space that is predicted to be the moduli space of E_{n+1} monopoles. Once again, the classical singularity is smoothed out by the world-sheet instanton corrections.

4.3 Parameter mapping

We conclude this Section by making the mapping of the Type I$'$ configuration discussed in Section 2 to the Heterotic set-up more explicit. We will give a more precise computation of the masses for electrically and magnetically charged objects.

The Type I′ background is given by [?, ?, ?]

$$\begin{aligned} e^{\phi} &\sim (\Omega(x_9)/C)^5 \\ g_{\mu\nu} &= \Omega^2(x_9)\eta_{\mu\nu} \end{aligned} \tag{12}$$

where $\Omega(x_9) \sim C^{5/6}(B + (8-n)x_9))^{-1/6}$, and where, for simplicity, all the π and α' factors have been ignored. Here x_9 belongs to the interval $[0, \pi]$.

The Type I′ background is specified by the parameters B and C. The relation with the Heterotic parameters R, ϕ_h is given by

$$Re^{-2\phi_h} \sim D^5 C^5$$
$$DC^{5/3} \sim (8-n)^{1/2}[(B + 2\pi(8-n))^{4/3} - B^{4/3}]^{-1/2}$$

where D^2 (a function of B and C) is the factor that converts the 9 dimensional Heterotic metric to the Type I′ metric.

The mass for a D0 brane stuck at the orientifold plane has been worked out and compared with the Heterotic BPS formula in [?, ?]. The result is

$$\frac{M_{D0}}{2} (=\sim \Omega(0)e^{-\phi(0)}) = \frac{1}{D}\left(\frac{R}{\alpha'} - \frac{(8-n)}{8R}\right) \tag{13}$$

The right hand side vanishes for $R = R_{\text{crit}}$: this tells us that the Type I′ electric objects that are becoming massless at the enhanced symmetry point are D0 branes stuck at the orientifold plane. $R - R^2_{\text{crit}}/R$ is identified with the E_{n+1} Higgs VEV in the Heterotic theory which enhances $SO(2n) \times U(1)$ to E_{n+1}. We see that the mass for a $W^{\pm}$ bosons is given by the Higgs VEV in agreement with the general expectations. The factor of 1/2 takes into accounts the fact that the D0 brane is stuck while D takes into accounts the rescaling between the Type I′ and the Heterotic metrics.

We can now do a similar check for a stuck NS brane. On general grounds, its mass should be given by the Higgs VEV divided by the square of the E_{n+1} coupling constant. The tension for a stuck NS brane is given by

$$T_{NS} \sim \int d^6x \frac{1}{g_s^2 l_s^6}\sqrt{g_I} \sim \Omega(0)^6 e^{-2\phi(0)} \tag{14}$$

Combining equations (12) with the first of equations (13), we can compute the ratio

$$\frac{1}{g_{YM}^2} = \frac{T_{NS}}{M_{D0}} = \Omega(0)^5 e^{-\phi(0)} \sim C^5 \sim \frac{1}{D^5} Re^{-2\phi_h} \tag{15}$$

The factor D^5 is just the effect of the metric rescaling. In Heterotic units, we recover the result that the E_{n+1} gauge coupling is $\frac{1}{g_{YM}^2} = Re^{-2\phi_h}$, in agreement with the perturbative analysis.

5 Smoothness of the moduli space

We expect the moduli space of monopoles to be a smooth manifold. All the previous examples were focused on configurations where we expected to get a smooth moduli space. As discussed in [?], due to the equation

$$\partial^2 \phi_h = \mathrm{Tr}\, F^2 - \mathrm{Tr}\, R^2 \tag{16}$$

singularities in $\mathrm{Tr}\, F^2$ drive the Heterotic string to a non-perturbative regime where we expect singularities in the moduli space. Singularities in $\mathrm{Tr}\, R^2$, on the other hand, keep the theory in the perturbative region and the only possible singularities may come from a break down of the two dimensional sigma model description. It was argued in [?] that in the absence of gauge fields the sigma-model can not fail. In [?, ?], the criterion for having a finite coupling constant was satisfied by considering no gauge fields at all. We considered generalizations where the gauge fields are present but still $\mathrm{Tr}\, F^2$ is regular. The background parameters, such as the radius of S^1 and the Wilson lines, were chosen in such a way that the 9 dimensional gauge group $SO(2n)$ is spontaneously broken to the Cartan sub-algebra. In this background, the only objects (that give rise to 6 dimensional moduli) introduced in the game were BPS monopoles of the space-time gauge group $SO(2n)$ and KK monopoles. BPS monopoles have a regular $\mathrm{Tr}\, F^2$ while KK monopoles may induce only a singularity in $\mathrm{Tr}\, R^2$.

It is fairly easy to consider singular configurations. Heterotic background with non trivial gauge fields associated with instantons suffer from small instanton singularities. Near the singular points, the perturbative description breaks down, since, due to equation (16), the dilaton is blowing up. The singularity survives quantum corrections and it is associated with a restoration of a six-dimensional gauge theory via Higgs mechanism. In the type I$'$ picture these configurations introduce extra D6 branes wrapped along S^1. When D6 branes touch, some degrees of freedom become massless and there is an enhanced gauge symmetry.

We can have more general non-perturbative vacua of the Heterotic string with 6 dimensional tensor multiplets. Their existence can be easily explained in the dual Type I theory, which can be thought as a Type II theory moded out by the world-sheet parity. Background with tensor multiplets can be easily obtained with a Z_k orbifold. In Type II, each of the $k-1$ twisted sectors give rise to a hypermultiplets and a tensor multiplet. The world-sheet parity may project out the tensor multiplet or the hypermultiplet. The various consistent possibilities are associated with the different types of gauge bundles, depending whether they admit vector structure or not. In such backgrounds, some of the blowing up modes of the ALE space are projected out, and the space-time can not become completely smooth. The configuration that we considered in this paper corresponds to projecting out all the tensor multiplets and it corresponds to the "compactification" on a smooth ALE space. The various 6 dimensional models that may be obtained in Type I orientifold constructions are discussed in [?, ?, ?]. The type I$'$ description was considered in [?, ?, ?]: the consistent models are obtained by disposing, in a Z_2 symmetric way, k NS branes on the segment. A NS brane in the middle of the segment supports a tensor multiplet, while a NS brane stuck at one of the orientifold points supports a hypermultiplet. D6 branes can be stretched between NS branes; they have the interpretation of small fractional instantons in the Heterotic string. The number of D6 branes is fixed by RR space-time charge conservation; as shown in [?], this is completely equivalent to the anomaly cancellation in the six-dimensional gauge theory. All these moduli space have small instanton singularities at the origin of the Higgs branch.

There is a natural mapping of all these six-dimensional configurations to three-dimensional $N=4$ gauge theories, where mirror symmetry can be used in order to extract information on the moduli space. For example, it was explained in [?], using the brane description, how the moduli space of n small E_8 instantons on an A_k singularity is mapped to the Coulomb branch of a three dimensional, $N=4$ supersymmetric, $U(k)$ gauge theory with n flavors.

The configuration with all the NS branes stuck at one of the orientifolds was considered in [?] only for a specific example. We see that it generically represents the Type I$'$ description of the Type I or Heterotic vacuum corresponding to a "compactification" on a smooth ALE space.

6 D_n singularities

Let us briefly consider the case of D_k singularities.

From the Heterotic point of view, we consider the moduli space of D_k ALF spaces. These space are asymptotic to $(R^3 \times S^1)/D_k$, where D_k is the binary dihedral group of order $4(k-2)$.

Unlike their cousins - the A_k ALF spaces - that have the very simple metric (7) with the potential (8), the D_k ALF space metric is much more complicated [?, ?, ?, ?]. Asymptotically, for large $|x|$, their metric can be given in the form of equation (7) with a potential

$$V(\vec{x}) = 1 - \frac{4}{|x|} + \sum_{i=1}^{k} \frac{1}{|x - x_i|} + \sum_{i=1}^{k} \frac{1}{|x + x_i|} \tag{17}$$

Some explicit construction for the full metric can be found in [?, ?, ?].

Consider, for simplicity, a Heterotic background without gauge fields. When we tune the radius R to the self-dual point, we still expect to find an $SU(2)$ symmetry in the spectrum. With respect to the Taub-NUT space, the D_k ALF spaces have an additional Z_2 action that combines with the cyclic group C_k to produce the dihedral group D_k. This Z_2 is manifest in equation (17) and acts non-trivially on the space R^3. As a result, the nine-dimensional $SU(2)$ gauge theory is moded out by a Z_2 that changes sign to three of the nine space-time coordinates. The nine-dimensional Lorentz invariance is explicitly broken; we are dealing with an orbifold gauge theory, or, in other words, with a theory with impurities. The moduli space of D_k ALF spaces is then identified with the moduli space of k monopoles of the orbifolded $SU(2)$ gauge theory. We can easily realize the same monopole configuration in terms of branes; for example, with D3 branes stretched between two NS branes in the presence of an orientifold plane O3$^-$. The corresponding monopole moduli space is then identified with the Coulomb branch of an N=4 three-dimensional $SO(2k)$ gauge theory by using the standard rules from [?]. The relation between the moduli space for Heterotic on D_k ALE singularities and the Coulomb branch of this gauge theory has been conjectured in [?].

The Type I$'$ description is simple. As discussed in [?], a D_k singularity is modeled in the dual picture using NS branes in the presence of an ON^0 plane. There are two ON^0 planes, each at the endpoints of the Type I$'$ interval. The presence of an O8$^-$ plane induces in addition an O6 plane.

There are six-dimensional fields living on the ON^0 planes; if we choose an O6$^-$ plane, there is a hypermultiplet on the ON^0 plane [**?**].

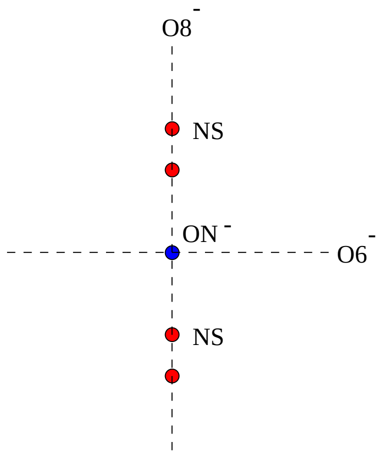

Figure 3: D_k monopoles is Type I$'$. The vertical dashed line represents an $O8^-$ plane. The horizontal dashed line represents an $O6^-$ plane. At the intersection of both there is an ON^- plane. $2k$ half NS branes are placed around the ON plane in a symmetric fashion. As for the A_k case half NS branes are confined to the $O8^-$ plane.

We will consider only one of the ON^0 planes, as the physics is confined to one of the $O8^-$ planes. It is more convenient, for our purposes, to think of an ON^0 plane as the combination of a ON^- plane with a physical NS brane. We can put extra $2k-2$ stuck NS branes on the O8$^-$, as in figure 3. We think of this system as an ON^- plane with $2k$ half NS branes stuck to the $O8^-$ plane [**?**]. Since there is an O6$^-$ plane, each of the half NS branes have an image on the O8$^-$ plane. We have a total of k hypermultiplets that parameterize our moduli space. There is a cosmological constant in the bulk that prevents the NS branes from leaving the orientifold plane. As a consequence, there is no singularity associated to their motion in the bulk and the moduli space is smooth.

There is a potential weak point in this argument due to the fact that two half NS branes in the game arise in a perturbative description of the ON^0 plane as twisted states. The difficulty stems from the fact that we do not have a satisfying description for the ON^- plane whereas the ON^0 plane admits a perturbative description. The two half NS branes related to the ON^0 plane however do not leave the $O8^-$ plane by the same argument of charge conservation valid for the other NS branes. As an example, we can consider the case $k = 2$. The Heterotic dual contains a D_2 singularity. Since D_2 is the product of two disjoint A_1 singularities, we expect that the moduli space is the product of two Atiyah-Hitchin manifolds. For $k = 1$ the group D_1 is $SO(2)$ and the configuration is just an $ON^0(= ON^- +$ NS) plane located at the intersection of the $O8^-$ and $O6^-$ planes. The moduli space is flat space as expected from half a NS brane and its image as they leave the ON^- plane.

In this way, the Type I′ picture suggests that the moduli space of orbifolded gauge theories is smooth despite the singularity in the space where the monopoles are living. We can also formulate a related conjecture for three-dimensional gauge theories: the Coulomb branch of N=4 SYM $SO(2k)$ theories is a smooth manifold.

The generalization to include D8 and D6 branes is straightforward.

7 Conclusions

We provided an explicit construction of E_{n+1} monopoles in string theory. Our investigation leads us to identify the monopoles as Half NS branes stuck at an O8 plane in Type I′. These NS branes transform in the spinor representation of an $SO(2n)$ subgroup of E_{n+1}. Conservation of RR charge does not allow the NS branes to move outside the orientifold plane, giving a stringy interpretation for the smoothness of the monopole moduli space. The same argument can be used to predict smoothness in the moduli space of more general and less studied gauge theory monopoles.

We also connected our configuration with Heterotic backgrounds where monopole naturally appear in the form of KK monopoles. We were able to give an explanation and to generalize some results on the Heterotic moduli space appeared in the literature [**?**, **?**]. We did not discuss the appearance of three-dimensional gauge theories in this context as they are very natural in view of our discussion and the results of [**?**, **?**].

In this paper, we just considered an example of a particular class of monopoles, which, nevertheless, has many ramifications and connections with different string backgrounds. Many other monopole configurations should naturally show up in string theory. We already know of monopole moduli spaces appearing in gauge and string theories in many contexts, stemming from three-dimensional gauge theories to singular Heterotic backgrounds.

Every time an Atiyah-Hitchin manifold shows up in the string moduli space, it is worthwhile to look around for monopoles.

A natural direct extension of our investigation would be to study systems with D7 branes. Exceptional groups also appear quite naturally in this context. A chain of dualities would lead us to consider more general Heterotic backgrounds and F-theory models.

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